Towards a determination of the chiral couplings at NLO in $1/N_C$: $L_8^r(\mu)$

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We present a dispersive method which allows to investigate the low-energy couplings of chiral perturbation theory at the next-to-leading order in the $1/N_C$ expansion, keeping full control of their renormalization scale dependence. As an example we determine the value of L_8 at $\mu_0 = 0.77$ GeV to be $L_8^r(\mu_0)^{SU(3)} = (0.6 \pm 0.4) \cdot 10^{-3}$.

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THE LARGE–N_C LIMIT

In recent years we have witnessed a spectacular progress in our understanding of low-energy effective field theories [1, 2, 3]. In particular, chiral perturbation theory (χPT) has been established as a very powerful tool to incorporate the chiral symmetry constraints when analysing the strong interactions in the non-perturbative regime [4, 5, 6]. The precision required in present phenomenological applications makes necessary to include corrections of $\mathcal{O}(p^6)$. While many two-loop χPT calculations have been already performed, the large number of unknown low-energy couplings (LECs) appearing at this order puts a clear limit to the achievable accuracy [7].

The limit of an infinite number of quark colours has proved very useful to bridge the gap between χPT and the underlying QCD dynamics [8, 9]. Assuming confinement, the strong dynamics at $N_C \to \infty$ is given by tree diagrams with infinite sums of hadron exchanges, which correspond to the tree approximation of some local effective Lagrangian [10, 11]. Resonance chiral theory (RχT) provides the correct framework to incorporate the massive mesonic states within an effective Lagrangian formalism [12]. Integrating out the heavy fields one recovers the χPT Lagrangian with explicit values of the chiral LECs in terms of resonance parameters. Moreover, the short-distance properties of QCD impose stringent constraints on the low-energy parameters [13].

Truncating the infinite tower of meson resonances to the lowest states with 0^{-+}, 0^{++}, 1^{--} and 1^{++} quantum numbers (single-resonance approximation, SRA), one gets a very successful prediction of the $\mathcal{O}(N_C p^4)$ χPT couplings in terms of only three parameters: M_V, M_S and the pion decay constant F [8]. Some $\mathcal{O}(p^6)$ LECs have been already predicted in this way, by studying an appropriate set of three-point functions [14]. More recently, the programme to determine all $\mathcal{O}(p^6)$ LECs at leading order in $1/N_C$ has been put on very solid grounds, with a complete classification of the needed terms in the RχT Lagrangian [15].

Since chiral loop corrections are of next-to-leading order (NLO) in the $1/N_C$ expansion, the large–N_C determination of the LECs is unable to control their renormalization-scale dependence. For couplings related with the scalar sector this introduces large uncertainties, because their μ dependence is very sizeable. A first analysis of resonance loop contributions to the running of $L_{10}^r(\mu)$ was attempted in Ref. [16]. More recently, a NLO determination of the χPT coupling $L_9^r(\mu)$ has been achieved, through a one-loop calculation of the vector form factor in RχT [17]. In spite of all the complexity associated with the still not so well understood renormalization of RχT [17, 18], this calculation has shown the potential predictivity at the NLO in $1/N_C$.

In this letter we present a NLO determination of the coupling $L_8^r(\mu)$. Using analyticity and unitarity we can avoid all technicalities associated with the renormalization procedure, reducing the calculation to tree-level diagrams plus dispersion relations. This allows to understand the underlying physics in a much more transparent way. In particular, the subtle cancelations among many unknown renormalized couplings found in Ref. [17] and the relative simplicity of the final result can be better understood in terms of the imposed short-distance constraints.

Let us consider the two-point correlation functions of two scalar or pseudoscalar currents, in the chiral limit. Of particular interest is their difference $\Pi(t) \equiv \Pi_S(t) - \Pi_P(t)$, which is identically zero in QCD perturbation theory. When $t \to \infty$, this correlator vanishes as $1/t^2$, with a coefficient proportional to $\alpha_s \langle \bar{q}\Gamma q\, \bar{q}\Gamma q\rangle$ [19, 20]. The low-momentum expansion of $\Pi(t)$ is determined by χPT to have the form [4]

$$\Pi(t) = B_0^2 \left\{ \frac{2F^2}{t} + 32 L_8^r(\mu) + \frac{\Gamma_8}{\pi^2}\left(1 - \ln\frac{-t}{\mu^2}\right) + \mathcal{O}\left(t\right)\right\}, \tag{1}$$

with $\Gamma_8 = 5/48$ [3/16] in the $SU(3)_L \otimes SU(3)_R$ [$U(3)_L \otimes U(3)_R$] effective theory. Since $\Pi(t)$ does not depend on the χPT scale μ, the coefficient of the logarithm determines the scale dependence of the renormalized chiral coupling: $\mu \frac{dL_8^r(\mu)}{d\mu} = -\Gamma_8/(16\pi^2)$. The correlator is proportional to $B_0^2 \equiv \langle \bar{q}q\rangle^2/F^4$, which guarantees the right dependence with the QCD renormalization scale. The couplings F^2 and L_8 are $\mathcal{O}(N_C)$, while Γ_8 is $\mathcal{O}(1)$ and represents a NLO effect.

In the large–N_C limit, $\Pi(t)$ can be easily computed within RχT:

$$\Pi(t) = 2\,B_0^2\left\{\sum_i \frac{8\,c_{m_i}^2}{M_{S_i}^2-t} - \sum_i \frac{8\,d_{m_i}^2}{M_{P_i}^2-t} + \frac{F^2}{t}\right\}. \tag{2}$$

For a finite number of resonances, one finds that imposing the right high-energy behaviour ($\sim 1/t^2$) constrains the resonance parameters to obey the relations:

$$\sum_i\left(c_{m_i}^2 - d_{m_i}^2\right) = \frac{F^2}{8}, \qquad \sum_i\left(c_{m_i}^2 M_{S_i}^2 - d_{m_i}^2 M_{P_i}^2\right) = \widetilde{\delta}, \tag{3}$$

where $\widetilde{\delta} \equiv 3\,\pi\alpha_s F^4/4 \approx 0.08\,\alpha_s F^2 \times (1\ \text{GeV})^2$. Truncating the infinite sums to their first contributing states and neglecting $\widetilde{\delta}$, these relations fix the corresponding scalar and pseudoscalar couplings in terms of the resonance masses:

$$c_m^2 \;=\; \frac{F^2}{8}\,\frac{M_P^2}{M_P^2-M_S^2}\,, \qquad d_m^2 \;=\; \frac{F^2}{8}\,\frac{M_S^2}{M_P^2-M_S^2}\,. \tag{4}$$

Note that Eq. (4) imposes $M_P \geq M_S$. On the other hand, the low-energy expansion of (2) determines

$$L_8 = \sum_i\left\{\frac{c_{m_i}^2}{2\,M_{S_i}^2} - \frac{d_{m_i}^2}{2\,M_{P_i}^2}\right\} \approx \frac{F^2}{16\,M_S^2} + \frac{F^2}{16\,M_P^2}\,. \tag{5}$$

Using the approximate constraint $M_P/\sqrt{2} \approx M_S \sim 1$ GeV [21], this gives $L_8 \approx 3F^2/(32M_S^2) \approx 0.8\cdot 10^{-3}$. However, one does not known at which scale μ this prediction applies.

NLO CORRECTIONS

At the NLO in $1/N_C$, $\Pi(t)$ has a contribution from one-particle exchanges, with the structure in Eq. (2), plus one-loop corrections $\Delta\Pi(t)$ generating absorptive contributions from two-particle exchanges. The corresponding spectral functions of the scalar and pseudoscalar correlators take the form:

$$\frac{1}{\pi}\text{Im}\Pi_S(t) \;=\; 2B_0^2\left\{8\,c_m^2\delta(t-M_S^2) + \frac{3\,\rho_S(t)}{16\pi^2}\right\}, \tag{6}$$

$$\frac{1}{\pi}\text{Im}\Pi_P(t) \;=\; 2B_0^2\left\{F^2\delta(t) + 8\,d_m^2\delta(t-M_P^2) + \frac{3\,\rho_P(t)}{16\pi^2}\right\},$$

with

$$\begin{aligned}\rho_S(t) \;=&\; \frac{\theta(t)}{2}\,|F_S^{\pi\pi}(t)|^2 + \theta(t-M_P^2)\left(1-\frac{M_P^2}{t}\right)|F_S^{P\pi}(t)|^2 \\ &+\;\theta(t-M_A^2)\frac{t^2}{4M_A^4}\left(1-\frac{M_A^2}{t}\right)^3|F_S^{A\pi}(t)|^2+\cdots\end{aligned} \tag{7}$$

$$\begin{aligned}\rho_P(t) \;=&\; \theta(t-M_V^2)\frac{t^2}{4M_V^4}\left(1-\frac{M_V^2}{t}\right)^3|F_P^{V\pi}(t)|^2 \\ &+\;\theta(t-M_S^2)\left(1-\frac{M_S^2}{t}\right)|F_P^{S\pi}(t)|^2+\cdots\end{aligned} \tag{8}$$

We have adopted the single-resonance approximation and, moreover, we have only taken explicitly into account the lowest-mass two-particle exchanges: two Goldstone bosons or one Goldstone and one heavy resonance. In the energy region we are interested in, exchanges of two heavy resonances or higher multiplicity states are kinematically suppressed. Our normalization takes into account the different flavour-exchange possibilities. The relevant two-particle cuts are governed by the following scalar,

$$\begin{aligned}F_S^{\pi\pi}(t) \;&=\; \frac{M_S^2}{M_S^2-t}\,,\\ F_S^{P\pi}(t) \;&=\; \sqrt{1-\frac{M_S^2}{M_P^2}}\,\frac{M_SM_P}{M_S^2-t}\,,\\ F_S^{A\pi}(t) \;&=\; 0\,,\end{aligned} \tag{9}$$

and pseudoscalar,

$$\begin{aligned}F_P^{V\pi}(t) \;&=\; -2\sqrt{1-\frac{M_V^2}{M_A^2}}\,\frac{M_V^2M_P^2}{(M_P^2-t)t}\,,\\ F_P^{S\pi}(t) \;&=\; \sqrt{1-\frac{M_S^2}{M_P^2}}\,\frac{M_S^2M_P^2}{(M_P^2-t)t}\,,\end{aligned} \tag{10}$$

form factors [22]. The RχT couplings generating these form factors have been determined imposing a good high-energy behaviour of the corresponding spectral functions, i.e. that the individual form factor contributions to $\rho_S(t)$ and $\rho_P(t)$ should vanish at infinite momentum transfer. Moreover, we have used the contraints (4) and the analogous relations (Weinberg sum rules and good high-energy behaviour of the vector form factor) emerging in the vector/axial-vector sector. It is quite remarkable that these short-distance constraints completely determine the form factors in terms of the resonance masses [22]. The form factor $F_S^{A\pi}(t)$ turns out to be identically zero, within the SRA.

Using its known analyticity properties, $\Delta\Pi(t)$ can be obtained from the spectral functions through a dispersion relation, up to a subtraction term which has the same structure as the tree-level scalar and pseudoscalar resonance exchanges [22]. Therefore, the unknown subtraction constants can be absorbed by a redefinition of

c_m, d_m, M_S and M_P at NLO in $1/N_C$:

$$\Pi(t) = 2B_0^2 \left\{ \frac{8\, c_m^{r\,2}}{M_S^{r\,2} - t} - \frac{8\, d_m^{r\,2}}{M_P^{r\,2} - t} + \frac{F^2}{t} + \Delta\Pi(t)|_\rho \right\} . \tag{11}$$

At large values of t, the one-loop contribution has the behaviour

$$\Delta\Pi(t)|_\rho = \frac{F^2}{t}\delta^{(1)}_{\rm NLO} + \frac{F^2 M_S^2}{t^2}\left(\delta^{(2)}_{\rm NLO} + \widetilde{\delta}^{(2)}_{\rm NLO} \ln\frac{-t}{M_S^2}\right) + \mathcal{O}\left(\frac{1}{t^3}\right). \tag{12}$$

Imposing the vanishing of the logarithm $\ln(-t)/t^2$ gives the constraint $\widetilde{\delta}^{(2)}_{\rm NLO} = 0$, leading to

$$\left(1 - \frac{M_V^2}{M_A^2}\right) = \frac{M_S^2}{M_P^2}\left(1 - \frac{M_S^2}{2\, M_P^2}\right) , \tag{13}$$

which requires $M_A \leq \sqrt{2} M_V$. Imposing the right short-distance behaviour ($\sim 1/t^2$) in $\Pi(t)$, one gets

$$\begin{aligned} F^2\,(1 + \delta^{(1)}_{\rm NLO}) - 8 c_m^{r\,2} + 8\, d_m^{r\,2} &= 0 , \\ F^2\, M_S^2\, \delta^{(2)}_{\rm NLO} - 8 c_m^{r\,2}\, M_S^{r\,2} + 8\, d_m^{r\,2}\, M_P^{r\,2} &= -8\,\widetilde{\delta} , \end{aligned} \tag{14}$$

where the corrections

$$\delta^{(m)}_{\rm NLO} = \frac{3M_S^2}{32\pi^2 F^2}\left\{ 1 + \left(1 - \frac{M_S^2}{M_P^2}\right)\xi^{(m)}_{S\pi} + 2\left(\frac{M_P^2}{M_S^2} - 1\right)\xi^{(m)}_{P\pi} - \frac{2M_P^2}{M_S^2}\left(1 - \frac{M_V^2}{M_A^2}\right)\xi^{(m)}_{V\pi} \right\} \tag{15}$$

are known functions of the resonance masses:

$$\begin{aligned} \xi^{(1)}_{S\pi} &= 1 - \frac{6M_S^2}{M_P^2} + \left(\frac{4M_S^2}{M_P^2} - \frac{6M_S^4}{M_P^4}\right)\ln\left(\frac{M_P^2}{M_S^2} - 1\right), \\ \xi^{(1)}_{P\pi} &= 1 + \frac{M_P^2}{M_S^2}\ln\left(1 - \frac{M_S^2}{M_P^2}\right), \\ \xi^{(1)}_{V\pi} &= 1 + \frac{3M_V^2}{M_P^2}\left[\frac{M_V^2}{M_P^2} - \frac{3}{2} + \left(1 - \frac{M_V^2}{M_P^2}\right)^2 \ln\left(\frac{M_P^2}{M_V^2} - 1\right)\right], \\ \xi^{(2)}_{S\pi} &= -4 + \left(2 - \frac{4M_S^2}{M_P^2}\right)\ln\left(\frac{M_P^2}{M_S^2} - 1\right), \\ \xi^{(2)}_{P\pi} &= 1 + \ln\left(\frac{M_P^2}{M_S^2} - 1\right), \\ \xi^{(2)}_{V\pi} &= \frac{M_P^2}{M_S^2}\left(1 - \ln\frac{M_S^2}{M_V^2}\right) - \frac{2M_V^2}{M_S^2}\left(1 - \frac{M_V^2}{M_P^2}\right) \\ &\quad + \left(\frac{M_P^2}{M_S^2} + \frac{2M_V^2}{M_S^2}\right)\left(1 - \frac{M_V^2}{M_P^2}\right)^2 \ln\left(\frac{M_P^2}{M_V^2} - 1\right) . \end{aligned} \tag{16}$$

$\mathbf{L_8^r}(\mu)$ AT NLO

The low-momentum expansion of the RχT correlator in Eq. (11) reproduces the $U(3)_L \otimes U(3)_R$ χPT result (1), with a definite prediction for the LEC $L_8^r(\mu)$:

$$\bar{L}_8^{U(3)} \equiv \left[L_8^r(\mu) + \frac{\Gamma_8}{32\pi^2}\ln\frac{\mu^2}{M_S^2}\right]_{U(3)} = \frac{F^2}{16}\left(\frac{1}{M_S^{r\,2}} + \frac{1}{M_P^{r\,2}}\right) \times \left\{ 1 + \delta^{(1)}_{\rm NLO} - \frac{M_S^{r\,2}\delta^{(2)}_{\rm NLO} + 8\widetilde{\delta}/F^2}{M_S^{r\,2} + M_P^{r\,2}} \right\} - \frac{3\,\Delta}{256\pi^2} , \tag{17}$$

with

$$\begin{aligned} \Delta = 1 &- \left(1 - \frac{M_V^2}{M_A^2}\right)\left[\frac{17}{6} - 7\frac{M_V^2}{M_P^2} + 4\frac{M_V^4}{M_P^4}\right] \\ &+ \left(1 - \frac{M_V^2}{M_A^2}\right)\left(1 - \frac{4M_V^2}{M_P^2}\right)\left(1 - \frac{M_V^2}{M_P^2}\right)^2 \ln\left(\frac{M_P^2}{M_V^2} - 1\right) \\ &+ \left(\frac{M_P^2}{M_S^2} - 1\right)\left[2 + \left(\frac{2M_P^2}{M_S^2} - 1\right)\ln\left(1 - \frac{M_S^2}{M_P^2}\right)\right] \\ &+ \left(1 - \frac{M_S^2}{M_P^2}\right)\left[\frac{1}{6} + \frac{M_S^2}{M_P^2} - 4\frac{M_S^4}{M_P^4}\right] \\ &+ \frac{M_S^4}{M_P^4}\left(1 - \frac{M_S^2}{M_P^2}\right)\left(3 - \frac{4M_S^2}{M_P^2}\right)\ln\left(\frac{M_P^2}{M_S^2} - 1\right) . \end{aligned} \tag{18}$$

We have used the relations in Eq. (14) to eliminate the explicit dependence on the effective couplings c_m^r and d_m^r.

Eq. (17) modifies the large-N_C result in (5) with NLO corrections $\delta^{(1)}_{\rm NLO}$, $\delta^{(2)}_{\rm NLO}$ and Δ, which are fully known in terms of resonance masses. Moreover, our calculation has generated the right renormalization-scale dependence, giving rise to an absolute prediction for the scale-independent parameter $\bar{L}_8^{U(3)}$. Since we are working within the large-N_C framework, the Goldstone-nonet loops reproduce the non-analytic $\ln(-t)$ structure that arises in $U(3)_L \otimes U(3)_R$ χPT. To make contact with the usual $SU(3)_L \otimes SU(3)_R$ theory, we still need to integrate out the singlet η_1 field. Computing the massive one-loop η_1 contribution to $\Pi(t)$, one easily gets the known relation [23] between the corresponding L_8 couplings in the two chiral effective theories:

$$\begin{aligned} \bar{L}_8^{SU(3)} &= \bar{L}_8^{U(3)} + \frac{\Gamma_8^{SU(3)} - \Gamma_8^{U(3)}}{32\pi^2}\ln\frac{M_{\eta_1}^2}{M_S^2} \\ &= \bar{L}_8^{U(3)} - \frac{1}{384\pi^2}\ln\frac{M_{\eta_1}^2}{M_S^2} . \end{aligned} \tag{19}$$

The different input parameters are defined in the chiral limit. We take the ranges [4, 8, 24, 25, 26] $M_V = (770 \pm 5)$ MeV, $M_S^r = (1.14 \pm 0.16)$ GeV, $M_P^r = (1.3 \pm 0.1)$ GeV, $M_{\eta_1} = (0.85 \pm 0.05)$ GeV and $F = (89 \pm 2)$ MeV, and use the relation of Eq. (13) to fix M_A, keeping the constraint $M_P \geq M_S$ from Eq. (4) and imposing $M_A \geq 1$ GeV. The correction $\widetilde{\delta}$ turns out to be negligible. One obtains the numerical prediction

$$\bar{L}_8^{SU(3)} = (0.4 \pm 0.4) \cdot 10^{-3} . \tag{20}$$

The largest uncertainties originate in the badly known values of M_S^r and M_P^r, which already appear in the leading order prediction (5). To account for the higher-mass intermediate states which have been neglected in (6), we have added an additional truncation error equal to $0.12 \cdot 10^{-3}$, the contribution of the heaviest included channel ($P\pi$). All errors have been added in quadrature. At the usual χPT renormalization scale $\mu_0 = 0.77$ GeV, Eq. (20) implies

$$L_8^r(\mu_0)^{SU(3)} = (0.6 \pm 0.4) \cdot 10^{-3} \,, \tag{21}$$

to be compared with the value $L_8^r(\mu_0)^{SU(3)} = (0.9 \pm 0.3) \cdot 10^{-3}$, usually adopted in phenomenological analyses.

The sizeable numerical difference between $\bar{L}_8^{SU(3)}$ and $L_8^r(\mu_0)^{SU(3)}$ shows the large sensitivity of this coupling to the χPT renormalization scale. This is a general trend for those LECs which are dominated by scalar or pseudoscalar resonance exchanges. Therefore, to perform accurate phenomenological applications one needs to control the renormalization scale dependence, which requires a determination of the χPT couplings at NLO in $1/N_C$, like the one presented here for L_8.

The ideas discussed in this letter can be applied to generic Green functions, which opens a way to investigate other chiral LECs at NLO in the large-N_C expansion. Further work along these lines is in progress [22].

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